LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – **NOVEMBER 2013**

MT 4203 – ADVANCED MATHEMATICS FOR PHYSICS

Date : 12/11/2013 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

PART A

(Answer ALL the questions) **(10 X 2 = 20)**

1. Evaluate: $\int \frac{1}{9x^2-4}\,dx$.
2. Write down the formula for the Fourier values a_0 and a_n in the interval 0 to 2π.
3. Solve: $(D^2+5D+6)y=0$.
4. Verify that $(a^2-2xy-y^2)dx-(x+y)^2dy=0$ is exact.
5. Sketch the diagram to change the order of integration $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dxdy$.
6. Write down the transformation from Cartesian to polar co-ordinates.
7. Define irrotational field.
8. Prove that the vector $f=(x+3y)\vec{i}+(y-3z)\vec{j}+(x-2z)\vec{k}$ is solenoidal.
9. Define group.
10. Define contravariant tensor of order 2.

PART B

(Answer any Five questions) **(5 X 8 = 40)**

11. Evaluate: $\int x^3\cos 2x dx$.
12. Solve; $(D^2+D+1)y=x^2$.
13. Solve: $x\sqrt{1+y^2}+y\sqrt{1+x^2}\frac{dy}{dx}=0$.
14. Evaluate $\iint_R xy\,dx\,dy$, where R is the Region in the first quadrant bounded by the hypothesis $x^2-y^2=a^2$ and $x^2-y^2=b^2$ and the circles $x^2+y^2=c^2$ and $x^2+y^2=d^2$ $(0<a<b<c<d)$.
15. Change the order of integration in the integral $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}dxdy$ and evaluate it.
16. Explain any four fundamental operations in tensors.
17. Prove the following (i) $\nabla\cdot(r^3\mathrm{r})=6r^3$ (ii) *div* **r** =3.
18. Evaluate: $\int \frac{x+4}{6x-7-x^2}dx$.

PART C

(Answer any TWO questions) **(2 X 20 = 40)**

19. (a) Express $f(x) = x^2$ as Fourier series with period 2π, in the interval $(-\pi, \pi)$ and hence deduce that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots = \frac{\pi^2}{6}$$

(b) Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta) d\theta = \frac{\pi}{8} \log 2$. **(12+8)**

20. (a) Solve: $\left(D^2 + 4D + 5\right) y = e^x + \cos 2x$.

(b) Solve: $\frac{dy}{dx} = \frac{y^3 + 3x^2 y}{x^3 + 3xy^2}$. **(12+8)**

21. (a) Derive the relationship between Beta and Gamma functions.

(b) By transforming into spherical co-ordinates, Evaluate $\iiint xyz \; dx \, dy \, dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ **(12+8)**

22. (a) Verify Green's theorem in the XY plane for $\int_C \{(3x - 8y^2)dx + (4y - 6xy)dy\}$ where C is the boundary of the region given that $x = 0,\ y = 0,\ x + y = 1$.

(b) Define cyclic group and prove that every cyclic group is abelian. **(14+6)**

****